Business from technology

Adjoint-based uncertainty analysis of lattice-physics calculations with CASMO-4 (CRISTAL)

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March 19th, 2015

Outline

- Background and context
- Sensitivity and uncertainty (S&U) analysis
- Developed calculation system
- Examples of results

Background and Context

- *Reactor physics*: homogenized data for coupled neutronics/thermal-hydraulics calculations
- Ideally, all results should be accompanied by uncertainty estimates that characterize their plausibility
- In 2006, a benchmark was launched: Uncertainty Analysis in Best-Estimate Modelling (UAM) for Design, Operation and Safety Analysis of LWRs
  - *Objective:* propagate the uncertainty related to nuclear data through all stages in a coupled neutronics/thermal hydraulics calculation
  - *First stage:* develop S&U analysis methods for reactor physics codes
- At VTT, CASMO-4 is one of the standard tools for reactor physics calculations $\Rightarrow$ chosen as the S&U analysis platform

Sensitivity

- *Starting point:* mathematical model containing uncertain parameters and response dependent on this model
- *Question:* If one of the parameters is perturbed, how will this affect the response?
- *Mathematical definition:* Sensitivity of response R with respect to parameter α is the derivative

$$S = \frac{dR}{d\alpha} \tag{1}$$

- Application:

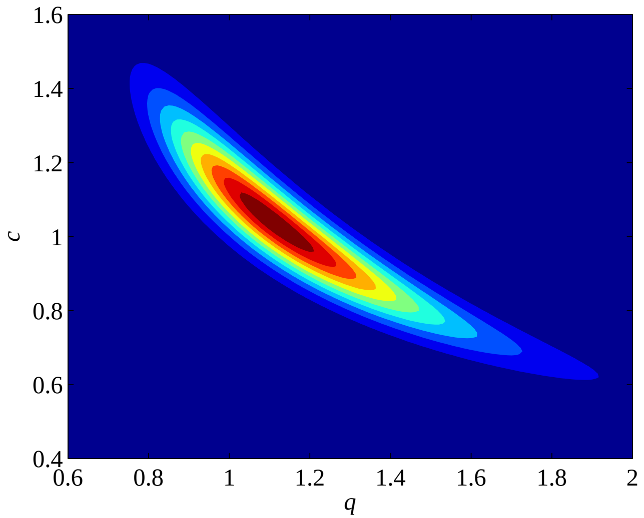
$$R \approx R(\hat{\alpha}) + S\delta\alpha \tag{2}$$

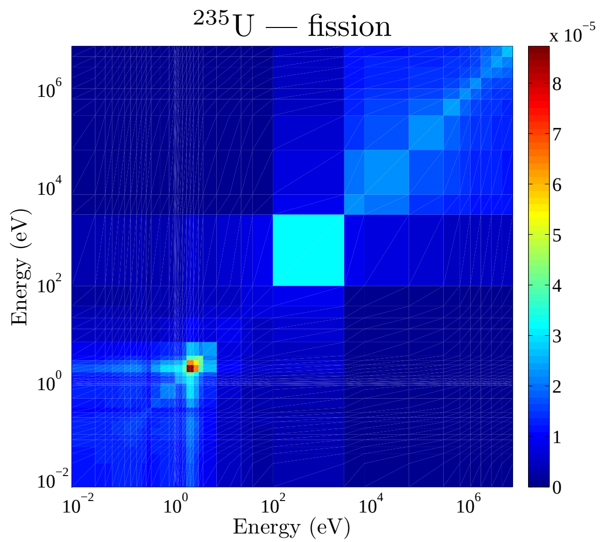
Sensitivity Analysis

- *Objective:* Compute derivatives with respect to all parameters of interest
- Brute-force approach:
  - Variate the parameters one-by-one and compute the response
  - Inefficient when there are several parameters
- Deterministic approach:
  - Formulate the problem mathematically and compute the derivatives
  - Very efficient if a mathematical concept called *adjoint* is utilized
  - In reactor physics, the adjoint solution can be interpreted as the importance of a neutron to the response

Uncertainty

- *Starting point:* a mathematical model containing uncertain parameters and a response dependent on this model
- Uncertainty related to the parameters can be presented as probability distributions
- Variance (one parameter) or covariance (several parameters) of the distribution may be chosen as the descriptive statistic for the uncertainty





Uncertainty Analysis

- *Objective:* Compute response variance due to uncertain parameters
- This is straightforward and simple if respective sensitivities are known
- Inaccuracy related to numerical methods or approximation errors not included in classical uncertainty analysis

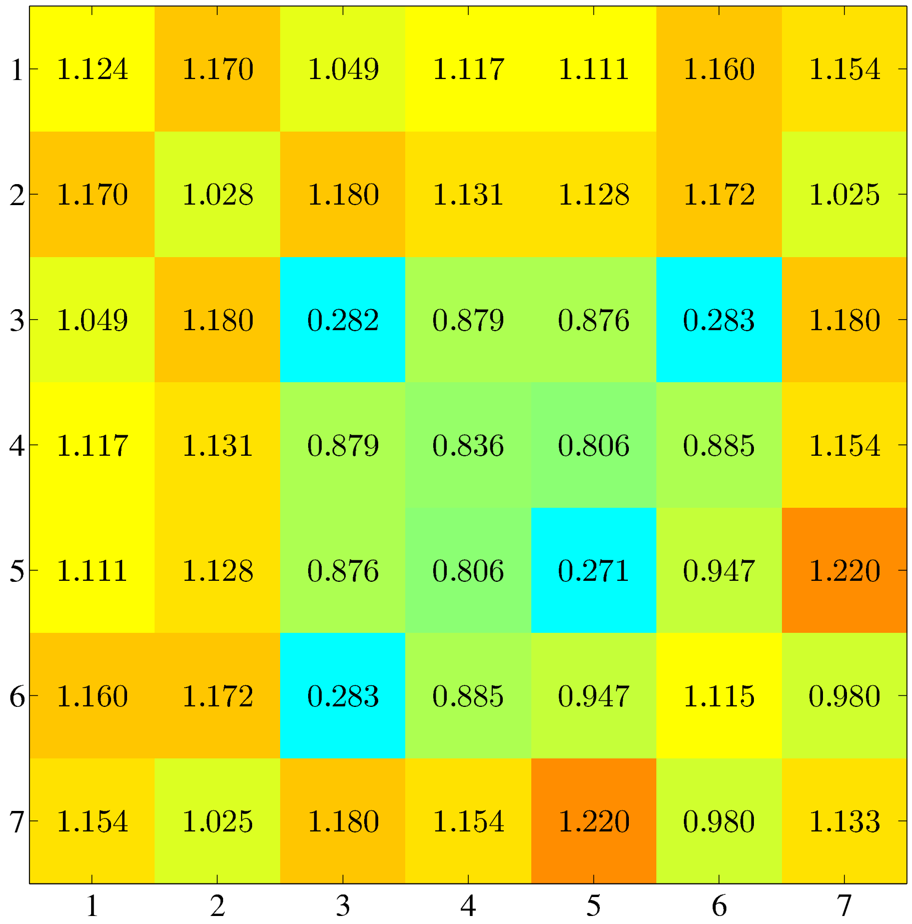
Application to Reactor Physics

- *Mathematical model:* criticality equation
- *Responses:* multiplication factor, homogenized few-group cross-sections, diffusion coefficients, assembly discontinuity factors

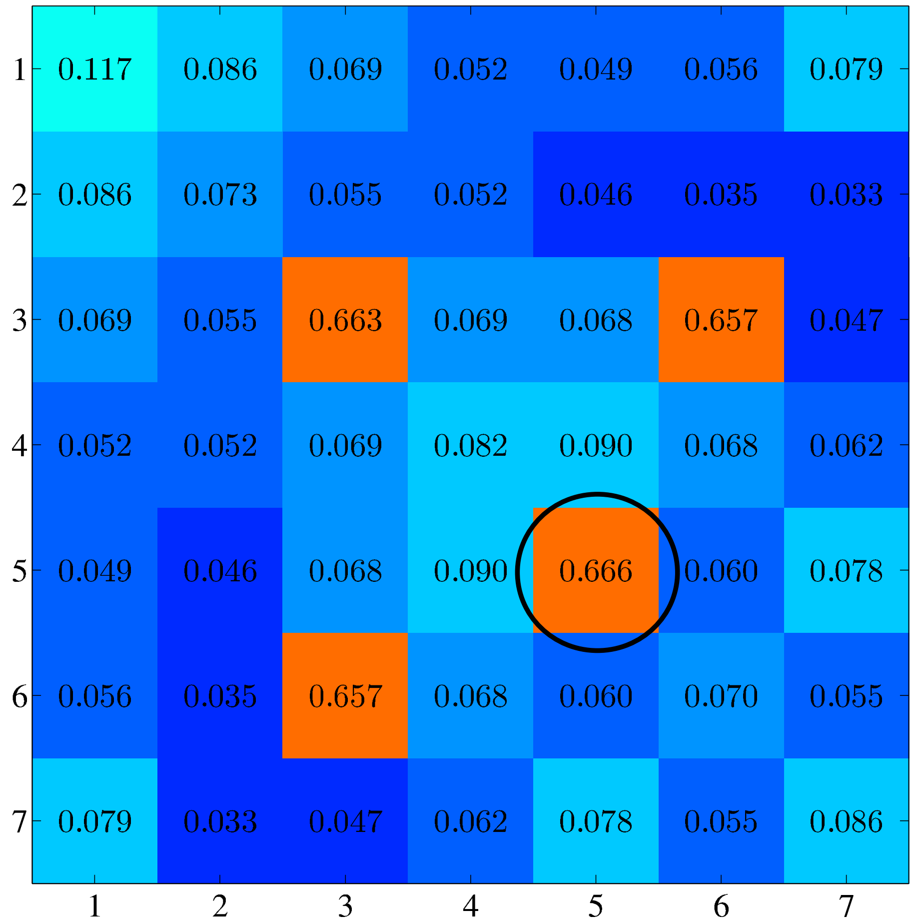
  $\Rightarrow$ Data passed on to nodal codes simulating a full core
- *Uncertain parameters:* nuclear data in the cross-section library of the code

  $\Rightarrow$ At least thousands of uncertain parameters
- Adjoint-based sensitivity and uncertainty analysis capability implemented to CASMO-4

Pin-power distribution for a BWR assembly:



Power distribution



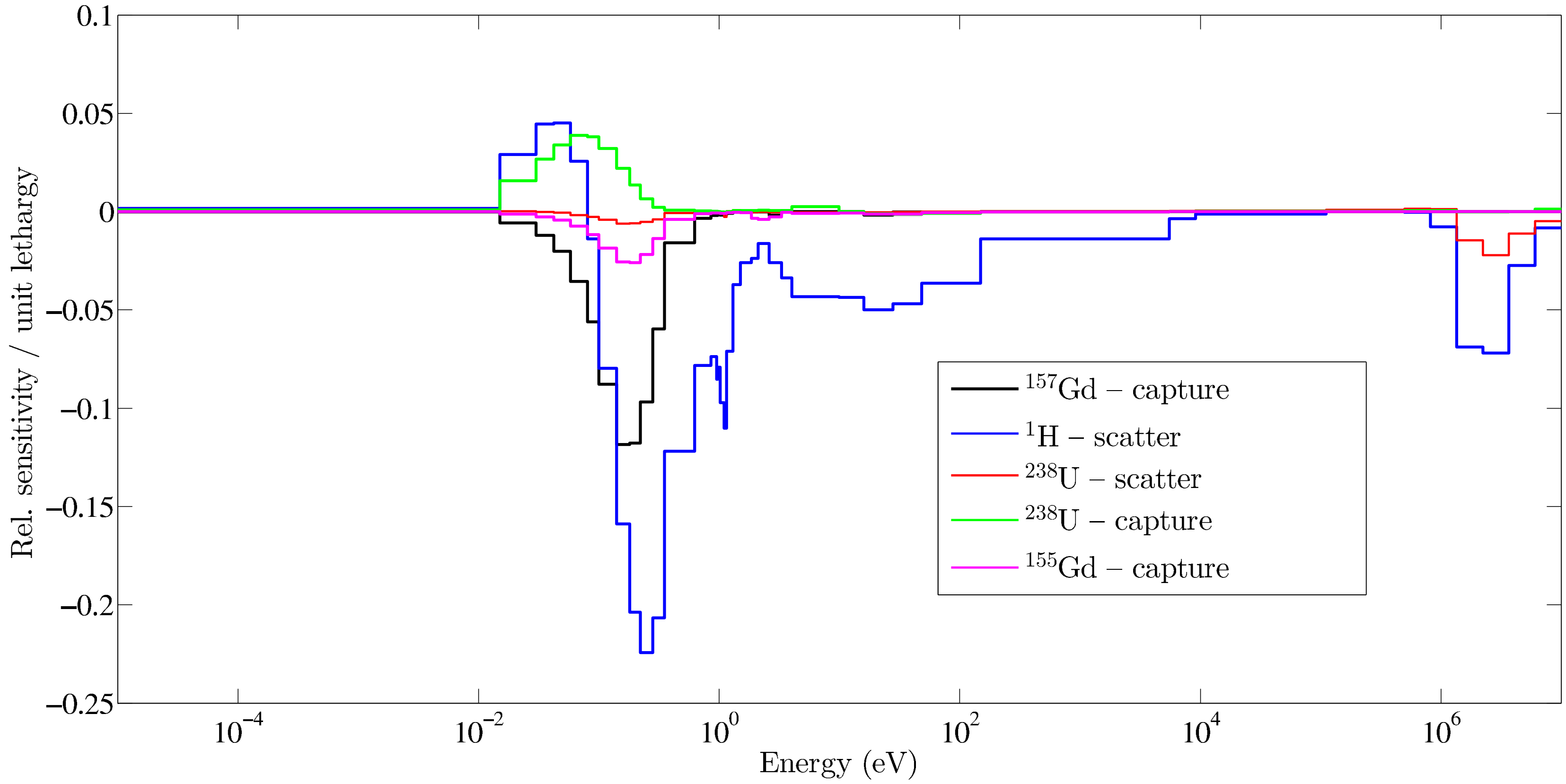
Relative uncertainty (%)

Uncertainty profile for pin with greatest uncertainty

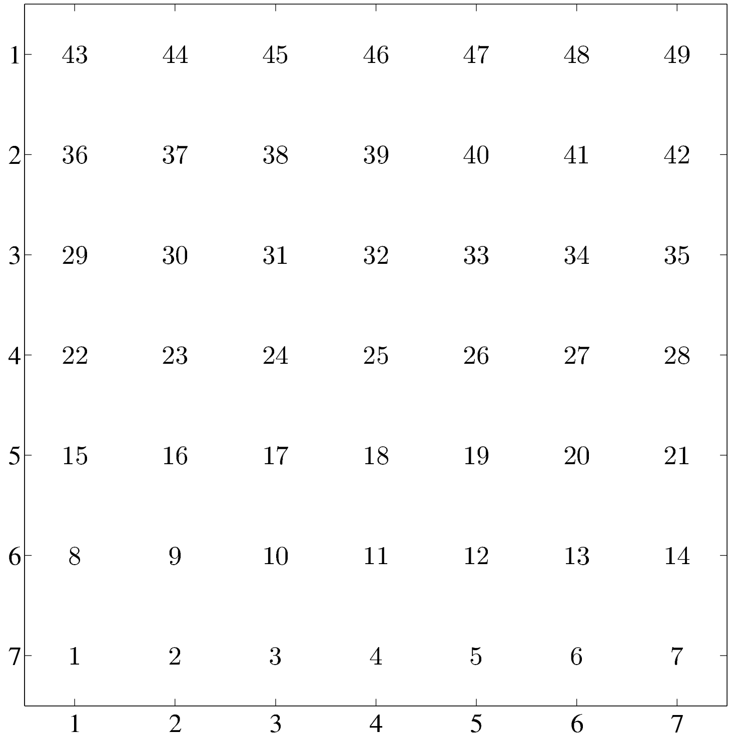
- Pin power: $R = 0.271$, Relative uncertainty: $\Delta R/R = 0.67$%

Nuclide	Param.pair	Rel. sensitivity	Contr. to $\Delta R/R$ (%)
^{235}U	$\chi,\ \chi$	-3.12$\times10^{-9}$	3.40$\times10^{-1}$
^{157}Gd	$\sigma_{\mathrm{c}},\ \sigma_{\mathrm{c}}$	-1.59$\times10^{-2}$	2.07$\times10^{-1}$
^{1}H	$\sigma_{\mathrm{s}},\ \sigma_{\mathrm{s}}$	-8.49$\times10^{-2}$	1.72$\times10^{-1}$
^{238}U	$\sigma_{\mathrm{s}},\ \sigma_{\mathrm{s}}$	-1.26$\times10^{-2}$	1.71$\times10^{-1}$
^{238}U	$\sigma_{\mathrm{c}},\ \sigma_{\mathrm{c}}$	-7.52$\times10^{-3}$	1.47$\times10^{-1}$
^{155}Gd	$\sigma_{\mathrm{c}},\ \sigma_{\mathrm{c}}$	-6.29$\times10^{-3}$	1.39$\times10^{-1}$
^{235}U	$\sigma_{\mathrm{c}},\ \sigma_{\mathrm{c}}$	-1.07$\times10^{-2}$	9.42$\times10^{-2}$
^{235}U	$\sigma_{\mathrm{f}},\ \sigma_{\mathrm{f}}$	-5.10$\times10^{-2}$	7.48$\times10^{-2}$
^{238}U	$\sigma_{\mathrm{f}},\ \sigma_{\mathrm{f}}$	1.32$\times10^{-2}$	6.88$\times10^{-2}$
^{1}H	$\sigma_{\mathrm{c}},\ \sigma_{\mathrm{c}}$	4.24$\times10^{-2}$	4.55$\times10^{-2}$

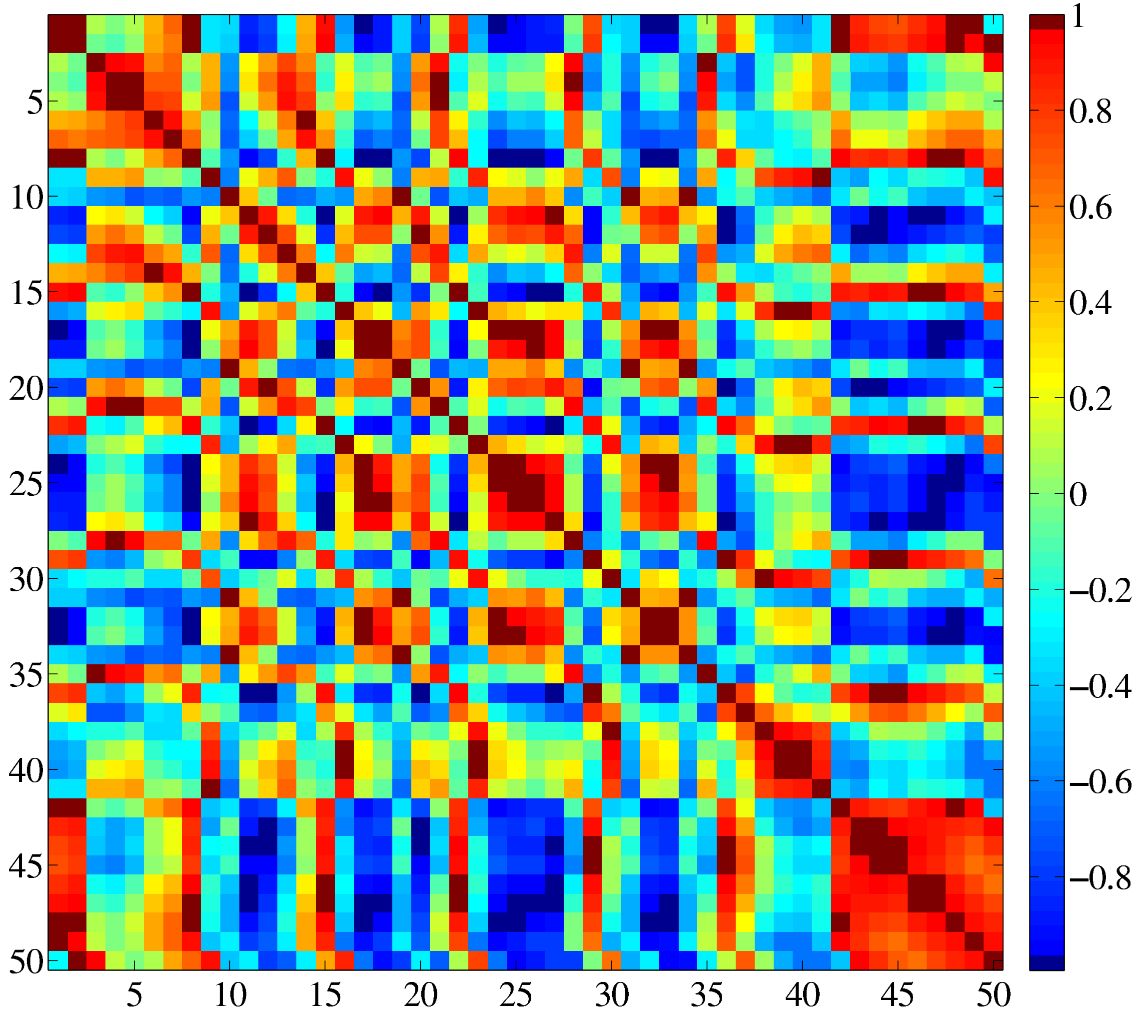
Sensitivity profiles for the same pin



Correlations of pin-power uncertainties



Numbering of pins



Correlation matrix for pin power uncertainties

Summary

- Uncertainty analysis: estimate the variance (or covariance) of calculation results due to uncertain parameters
- Straightforward and simple if sensitivities to parameters are known
- Sensitivities can be computed efficiently utilizing adjoint solutions
- Adjoint-based sensitivity and uncertainty analysis capability implemented to CASMO-4
- Enables the propagation of nuclear data uncertainty through assembly-level calculations used to produce data for full-core simulations